Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 11 MATHEMATICS

HSC ASSESSMENT 1

December 2007

TIME ALLOWED: 70 minutes

Instructions:

- Write your name and class at the top of this page, and at the top of each answer sheet.
- At the end of the examination this examination paper must be attached to the front of your answers.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- **START EACH QUESTION ON A NEW PAGE**

(FOR MARKERS USE ONLY)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL
/7	/6	/7	/6	/7	/7	/7	/7	/54

QUESTION 1: (7 MARKS)

(a) Evaluate $\sum_{r=1}^{4} \frac{1}{r(r+1)}$ **1**

(b) By considering $0.8\dot{3}$ as an infinite series (and by no other means), express this recurring decimal as a simplified fraction **2**

(SHOW ALL WORKING)

(c) The n[th] term of a sequence is given by $T_n = \frac{1}{3}(2^{n-1})$

(i) Find the values of the first, second and third terms

(ii) Prove that the series is geometric and find the common ratio **4**

(iii) Find the sum of the first 10 terms

QUESTION 2: (6 MARKS) START A NEW PAGE

Find derivatives of the following:

(a) $y = \frac{x^4}{4} + 2x + 3$ **2**

(b) $y = \sqrt{4 - x^2}$ **2**

(c) $y = \frac{3}{x-1}$ **2**

QUESTION 3: (7 MARKS) (START ON A NEW PAGE)

(a) For the sequence given by **4**

$$4, 7, 10, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.., 421$$

(i) Find an expression for the n^{th} term

(ii) Find the 45^{th} term

(iii) How many terms are there in the sequence?

(iv) Find the sum of all of the terms of this sequence

(b) Find the equation of the normal to the curve $y = x(x-2)(x-1)$ at the point where $x = 2$ **3**

QUESTION 4: (6 MARKS) (START ON A NEW PAGE)

(a) Find the value(s) of x for which the curve $y = x^2 - 3x + 1$ is increasing **2**

(b) Consider the series given by **4**

$$1 + 2x + 4x^2 + \ldots\ldots\ldots\ldots\ldots$$

(i) For what value(s) of x does this series have a limiting sum?

(ii) Assuming that the conditions on x in part (i) are satisfied, find the value of x for which the limiting sum is $\frac{4}{3}$

QUESTION 5: (7 MARKS) (START ON A NEW PAGE)

(a) A woman invests \$20 000 into a fund earning compound interest at 6% p.a.

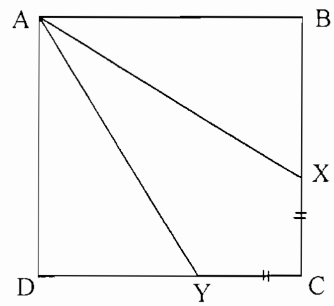
(i) Find the value of her investment after the first 6 years. **2**

(ii) After 6 years the compound interest rate rises to 8% p.a. and she leaves her money in the fund for a further 4 years. **1**
Find the value of her investment to the nearest dollar at the end of the10 years.

(b) In the diagram at right,

ABCD is a square

CX = CY



(i) Prove, giving all reasons, that $\Delta ABX \equiv \Delta ADY$ **3**

(ii) Hence prove that Δ AXY is isosceles **1**

QUESTION 6: (7 MARKS) (START ON A NEW PAGE)

(a) Frank invests \$250 per month into a superannuation fund, which earns interest of 12% per annum compounded monthly.

(i) What is the value of his investment at the end of 1 month? **1**

(ii) He continues to invest \$250 each month into the fund for 10 full years, the last payment being in the fund for a whole month. **3**

What is the total value of his investment after the 10 years?

(b) (i) Find the value of the derivative of the curve $y = x^3 + 3x^2 - 24x + 2$ at the point (2, -26). **2**

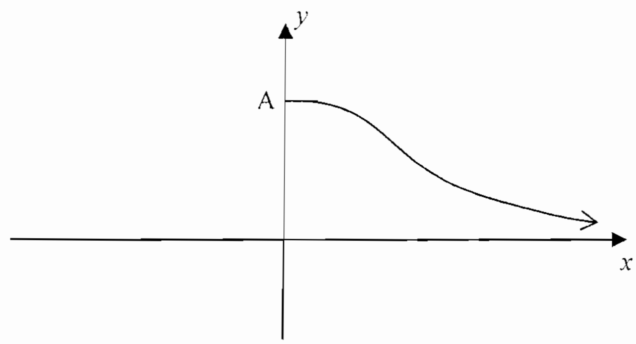
(ii) What is the geometric significance of this result with respect to the tangent to the curve at this point? **1**

QUESTION 7: (7 MARKS) (START ON A NEW PAGE)

(a) Consider the graph of $y = f(x)$ where $f(x) = \dfrac{1}{x^2 + 4}$

(i) Prove algebraically that this is an even function **1**

(ii) One half of the graph of $y = \dfrac{1}{x^2 + 4}$ is shown below: **2**



Find the coordinates of the point A in the diagram and draw the complete sketch of the curve on your answer sheet.

(b) (i) On a set of axes, draw the curve $y = |x - 5|$ showing all important features **2**

(ii) By drawing (and labelling) another line on this same set of axes, or otherwise, find all values of x for which **2**

$$|x - 5| \leq x + 5$$

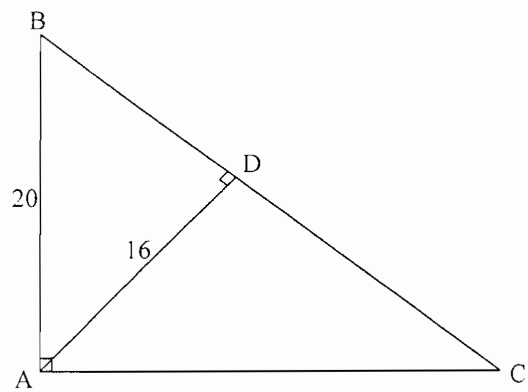
QUESTION 8: (7 MARKS) (START ON A NEW PAGE)

(a) Show that the curve $y = \dfrac{1-x}{2-x}$ is decreasing for all values of x **2**

(b) From the vertex A of the right angled triangle ABC shown at right, AD is drawn perpendicular to BC

AB=20 cm

AD= 16 cm



(i) Prove that ΔABC is similar to ΔDAC giving all reasons **2**

(ii) Find the length of BD **1**

(iii) It can also be shown that ΔABC is similar to ΔDBA. **2**
(YOU DO NOT HAVE TO PROVE THIS)

Using this result and parts (i) and (ii) above, find the length of DC

(END OF EXAMINATION)

2 UNIT SOLUTIONS

① (a) $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \frac{1}{4\times 5} = \frac{48}{60}$ ← ONLY THIS REQUIRED for 1 MARK.

$= \frac{4}{5}$

(b) $0.\dot{8}\dot{3} = 0.838383\ldots$

G.P. $a = 0.83$

$r = 0.01$

$S_\infty = \frac{0.83}{1-0.01}$ ← ① MARK

$= \frac{83}{99}$ ← ① MARK

(c) (i) $T_1 = \frac{1}{3}$ $T_2 = \frac{2}{3}$ $T_3 = \frac{4}{3}$ ← ① mark for all.

(ii) $T_2/T_1 = 2$ $T_3/T_2 = 2$ ← ① for SHOWING this

$r = 2$ ← ① MARK for stating

(iii) $S_{10} = \frac{\frac{1}{3}(1-2^{10})}{1-2}$ or $\frac{\frac{1}{3}(2^{10}-1)}{1}$

$= 1023/3$

$= 341$

} ① for any of these.

② (a) $dy/dx = x^3 + 2$ ← ① for each part = ②

(b) $dy/dx = \frac{1}{2}(-2x)(4-x^2)^{-1/2}$

$= \frac{-x}{\sqrt{4-x^2}}$

} ← ② but only ① if no negative

(c) $dy/dx = -3(x-1)^{-2}$

$= -\frac{3}{(x-1)^2}$

} EITHER for ② but only ① for no negative.

③ (a) (i) A.P. $a = 4$, $d = 3$

$T_n = 4 + (n-1)3$

$= 3n+1$ ← ① MARK

(ii) $T_{45} = 136$ ← ① MARK.

(iii) $3n+1 = 421$

$n = 140$ ← ① MARK.

(iv) $S_{140} = \frac{140}{2}(4+421) = 29750$

$= 140/2\,(8+139\times 3)$

} ① for any of these.

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(b) $\frac{dy}{dx} = 3x^2 - 6x + 2$ ← ① MARK

At $x = 2$ $m_T = 2$

$m_N = -\frac{1}{2}$ ← ① MARK.

$\therefore$ tangent is $y = -\frac{1}{2}(x-2)$

ie. $x + 2y - 2 = 0$

} (1

(4) (a) $dy/dx = 2x - 3$

For Increasing $dy/dx > 0$ ← (1) for showing this anywhere.

$\therefore \quad x > 3/2$ ← (1) MARK

(b) G.P. $a = 1$, $r = 2x$

(i) $-1 < 2x < 1$

$\therefore -1/2 < x < 1/2$ } ← (2) MARKS. Only (1) if only the solution $x < 1/2$ is given

NIL for $-1 < x < 1$

(ii) $S_\infty = 4/3$

$\therefore \frac{1}{1-2x} = 4/3$ ← (1) MARK

$\therefore 4 - 8x = 3$

$x = 1/8$ ← (1) MARK.

(5) (a) (i) $A_6 = 20\,000(1.06)^6$ ← (1)

$= 28,370.38$ ← (1).

$A_{10} = 20,000(1.06)^6(1.08)^4$

OR

$28,370.38(1.08)^4$

$= \$38,598.00$

} (1) only for answer.

(b) (i) In $\triangle ABX$ and $\triangle ADY$

AB = AD (sides of a square)

BX = DY (Since BC = DC, sides of a square and CX = CY, given)

$\angle ABX = \angle ADY$ (= 90°, angles in a square)

} (3)

$\therefore \triangle ABX \equiv \triangle ADY$ (SAS) ← subtract (1) if RHS

(ii) AX = AY (corresponding sides in congruent triangles)

$\therefore \triangle AXY$ is isosceles (2 sides equal) ← (1)

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(6) (a) (i) $A_1 = 250(1.01)$ ← ① mark for 0.01
$= 252.50$

(ii) $A_{12} = 250(1.01)^{120} + 250(1.01)^{119} + \ldots + 250(1.01)$

$= 250\left[\frac{1.01\left(1.01^{120}-1\right)}{1.01-1}\right]$ ① mark for ↑
① for 120

$= 58{,}084.77$ ① for answer

(b) (i) $\frac{dy}{dx} = 3x^2 + 6x - 24$ ← ① mark
At $(2, -26)$ $m_T = 0$ ← ① mark

(ii) The tangent is parallel to the x-axis ① mark

(7) (a) (i). $f(a) = \frac{1}{a^2+4}$

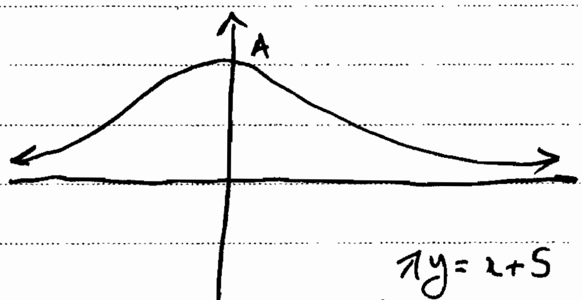
$f(-a) = \frac{1}{(-a)^2+4}$

$= \frac{1}{a^2+4}$

$= f(a)$.

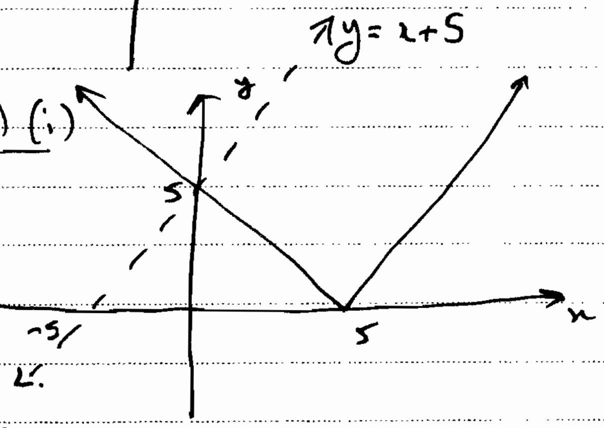
} (1) MARK. must show that $(-a)$ has been substituted.

(ii) A is $(0, \frac{1}{4})$ (1) MARK.



(1) for shape.

(b) (i)



(1) for shape

(1) for the two intercepts.

(ii) (1) MARK for the line $y = x+5$

(1) MARK for $x \geq 0$ (Don't penalise for $x > 0$)

OR

$x - 5 \leq x + 5$ or $x - 5 \geq -x - 5$ (1)

$-5 \leq 5$ $2x \geq 0$

NO SOLN $x \geq 0$ (1).

(8)

(a) $y = \frac{1-x}{2-x}$

$$\frac{dy}{dx} = \frac{(2-x)(-1)-(1-x)(-1)}{(2-x)^2}$$

$$= \frac{-2+x+1-x}{(2-x)^2}$$

$$= \frac{-1}{(2-x)^2}$$ ← (1) MARK

This is always negative for all x } (1) for a statement about
∴ Always decreasing. } the negativity of y'

(b) In △ABC and △DAC

$\angle BAC = \angle ADC = 90°$ (given) } (1) each reason
$\angle ACB$ is common. } = 2.
∴ △ABC ||| △DAC (equiangular) (1) MARK

(ii) $BD = \sqrt{20^2 - 16^2}$ (1) MARK.
$= 12$ cm

(iii) Since △ABC ||| △DBA (given)
and △ABC ||| △DAC (above)
∴ △DBA ||| △DAC.

∴ $\frac{AD}{BD} = \frac{DC}{AD}$

∴ $DC = \frac{AD^2}{BD}$
$= \frac{256}{12}$
$= 21\frac{1}{3}$ cm (1) MARK